Julia Robinson Mathematics Festival

23456 Calabasas Road, Suite 201
Calabasas, CA 91302
(408) 515-1908
jrmf.org

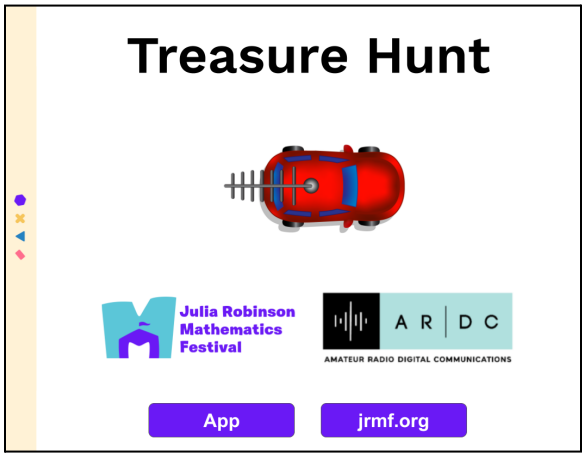
Building Mathematical Concepts Related to Radio Technology through Games, Manipulatives, and Discovery-Based Activities Final Report

Between April 2021 and December 2022, the Julia Robinson Mathematics Festival (JRMF) designed 3 activities related to ham radio and radio communication topics. For each activity, JRMF produced a set of activity slides with questions, tasks, and challenges along with a browser-based app built in Unity. To ensure that each activity was scientifically accurate and authentically represented the radio communication topics they were based on, JRMF consulted with a variety of ham club members as well as scientific literature. In depth information on the science behind the creation of each activity can be found in the Final Report Appendix.

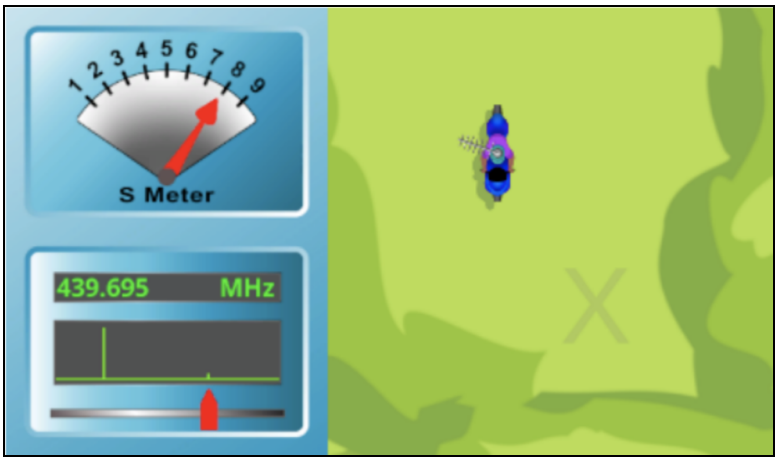
Each activity was playtested with a group of 60 students and received largely positive reviews. Based on feedback through student surveys, each activity was revised and polished to create the products included in this final report. Below we have included the resources created for each of the three activities – Treasure Hunt, Resistors, and Cryptography.

In addition to these three activities, the Julia Robinson Mathematics Festival hosted a free, public math festival on October 15, 2022 in collaboration with the local amateur radio group NASA-Ames Amateur Radio Club (AARC). Part of our ARDC grant was used as an honorarium for AARC to be present at our math festival and bring a variety of ham radio equipment. In addition to setting up a fully-functional ham radio, AARC also brought a collection of interactive ham radio activities that covered topics related to communications technology, like Morse code. Photos of AARC's contributions to the math festival are included below. The October 15 Math Festival had over 330 attendees (over 150 students) with a 97% satisfaction rating based on student surveys.

Activity #1: Treasure Hunt



Activity Slides



App

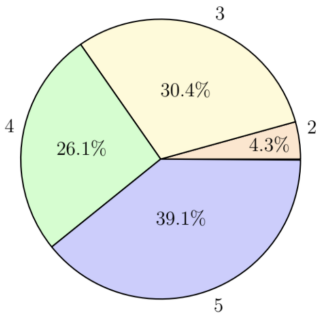
Treasure Hunt is an activity that emulates transmitter hunting, as popularized by the amateur radio community. The player drives a dirt bike scouting for buried treasure chests. Treasure is marked with an X and emits a radio signal that can be detected with an antenna and S meter. Students must develop strategies based on their knowledge of radio transmission technology to find each treasure. As students progress through this activity, they will encounter new challenges, like forests, mountains, and rivers that require them to adapt to their new environments in order to find the buried treasures.

Digital Communications Concepts:

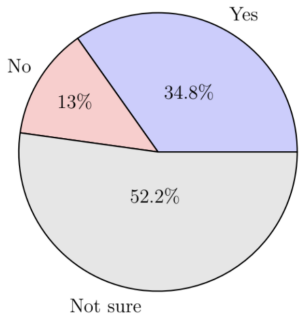
- A Yagi antenna with a realistic azimuth pattern
- Realistic S meter that can be overwhelmed by strong signals
- Ability to listen on either 146.565 MHz or its third harmonic
- Signal loss models taken from academic literature
- Mountains that reflect signals
- Background noise

60 students playtested Treasure Hunt. Based on student survey results, 65% of students enjoyed the activity and 35% expressed interest in participating in a real life transmitter hunt.

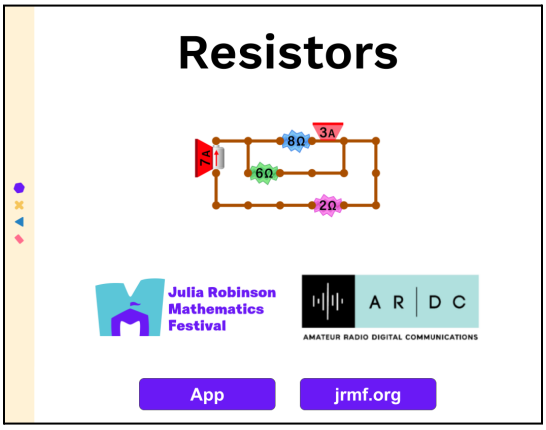
How much did you enjoy the activity? Scale: 1 (I did not like it at all) to 5 (I liked it a lot)



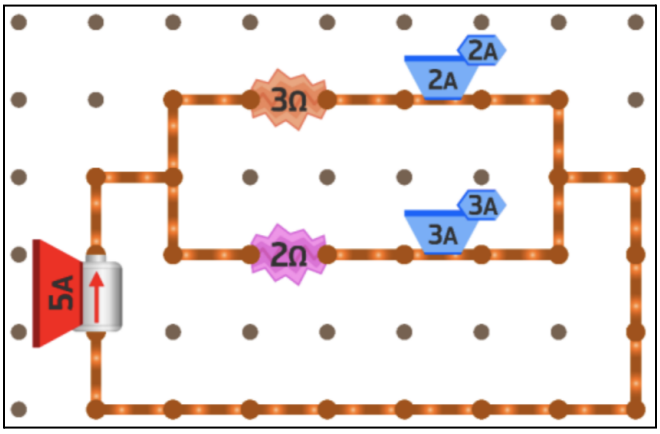
This activity was based on an activity people do outside with real radios. Do you think that is something you might like to try?



Activity #2: Transistors



[Activity Slides]



[App]

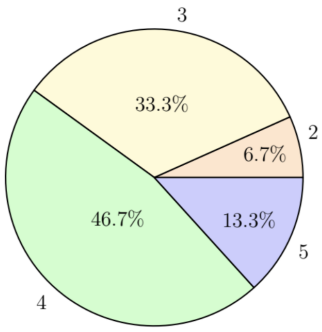
Resistors is an activity that allows students to explore the relationship between resistance, current, and voltage in a discovery-based way. This activity assumes no prior physics knowledge and instead provides an opportunity for students to experiment with a sequence of increasingly complex resistor-based puzzles. As students explore, they make, test, reject, revise, and verify hypotheses about the basic arithmetic of circuit design.

Digital Communications Concepts:

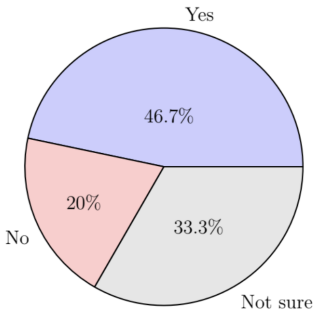
- Resistance
- Voltage
- Current
- Ohm's Law
- Parallel Circuits
- Series Circuits

60 students playtested Resistors. Based on student survey results, 60% of students enjoyed the activity and 47% expressed interest in electronic circuit design in real life.

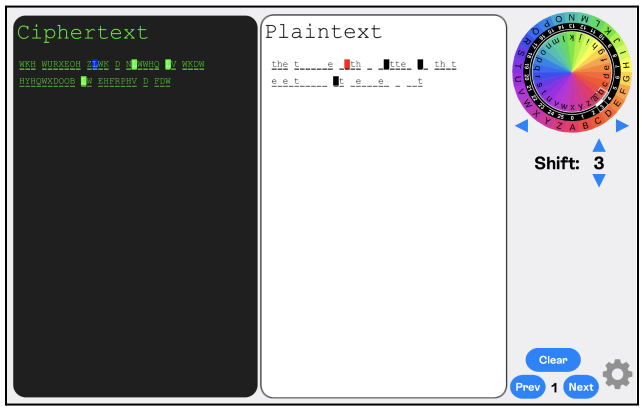
How much did you enjoy the activity? Scale: 1 (I did not like it at all) to 5 (I liked it a lot)



This activity was based on some principles of electronic circuit design, which people do both for fun and for work. Do you think that is something you might like to try?



Activity #3: Cryptography



[Activity Slides]

[App]

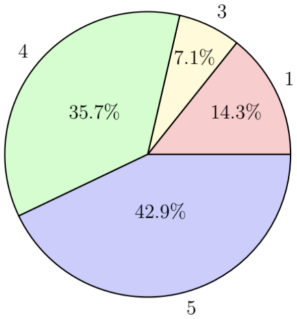
Cryptography is an activity that allows students to explore encryption and decryption through a variety of ciphers, including Caesar ciphers, Affine ciphers, Vigenère ciphers, and simple alphabetic substitution ciphers. In each puzzle, students decrypt a poem that is encrypted with an unknown cipher. The use of poetry was inspired by the Poetry in Motion campaign, during which poetry from a diverse authorship was displayed on public transit in various cities throughout the US. Many of the poems from that campaign are included in our Cryptography activity.

Digital Communications Concepts:

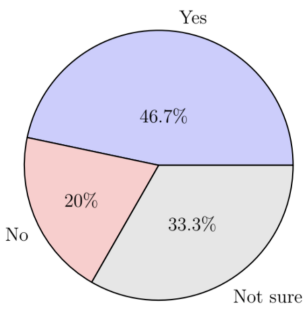
- Encryption
- Decryption
- Shifts
- Multipliers
- Caesar ciphers
- Affine ciphers
- Vigenère ciphers
- Simple alphabetic substitution ciphers

60 students playtested Cryptography. Based on student survey results, 79% of students enjoyed the activity and 64% expressed interest in further exploring cryptography.

How much did you enjoy the activity? Scale: 1 (I did not like it at all) to 5 (I liked it a lot)



This activity was based on some principles of electronic circuit design, which people do both for fun and for work. Do you think that is something you might like to try?



October 15 Math Festival

On October 15, 2022, JRMF hosted a free, open-to-the-public math festival in collaboration with the NASA-Ames Amateur Radio Club (AARC). Over 330 people attended the event, with over 150 students. Below are photos from the event.

Photos of minors removed for privacy by ARDC.

Learnings

The first major takeaway from our project was determining the average cost of creating an activity based on a ham radio topic. In our original grant application, we outlined a project that involved creating materials for two activities. By May of 2022, we realized that we had over budgeted for this project, and because of new staff members who were hired during the period of the grant, we were able to produce content more quickly and at a lower cost. For the first two activities, we would only spend about half of the grant budget ($63,500). With the additional funds from our grant, we proposed an amendment to our project that included a third activity as well as funding for our October 15 Math Festival. Through this project, we discovered that it takes approximately $27,000 on average to do the research for and create an activity based on a digital communications topic.

The second major takeaway from our project was identifying how to best conduct outreach and promote awareness for ham radio. We applied for this grant during the COVID lockdowns, and consequently, online and digital resources were our only way to connect with students and families at that time. As the lockdown restrictions eased, we were able to go back to hosting in person events, like the October 15 Math Festival. Although we enjoyed creating activities around digital communication topics, we believe that the math festival we hosted alongside AARC made a much bigger impact in spreading awareness about the ham radio community. During the creation of our ham radio-related activities, we found it challenging to connect with a large number of students and families. For much less time and money, we were able to reach about the same number of students and families through our math festival as we were able to through the outreach conducted using the activities we developed. As an added bonus, we were able to financially support a local ham radio group and promote the resources that they have to offer the community.

Based on these learnings, we believe that the most effective way to attract the next generation to the world of ham radio is through in person events, like our October 15 Math Festival. Although this first math festival took $20,000 to plan and implement, we believe that if we were to continue this project, we could cut down this cost to about $10,000 per festival and provide local ham radio groups a larger honorarium for their participation. During our planning of the October 15 Math Festival, we received interest from over a dozen ham radio groups in the Bay Area to be a part of such an event, and with the growing interest in in person math festivals over the past few months, we believe that there is a large demand for events like this one. With additional funding for the outreach portion of our project, we would be able to reach many more students in this area and show people the joy of both math and ham radio.

Final Budget

Description	**Cost**
Activity Research	$13,450
Activity Slides	$30,800
Activity Apps	$22,000
Activity Graphics	$13,750
Activity Teacher Resources	$2,000
Room and Hall Fees	$3,500
Event Supplies and Decorations	$1,500
Event Publicity	$3,500
Event Refreshments	$1,500
Event Personnel	$10,000
Administrative Fees	$10,200
Total	**$112,200**

ARDC-Funded JRMF Projects

Activities Report

Nick Rauh

January 10, 2023

Contents

Introduction

As part of our work funded by a grant from ARDC, JRMF has produced three math activities related to radio:

1. Treasure Hunt
2. Resistors
3. Cryptography

This report will give an overview of each activity, a look at some of the technical specifications, and notes on how the activity went when tested with students.

Chapter 1

Treasure Hunt

1.1 Activity overview

Treasure Hunt was inspired by transmitter hunting (or "fox hunting") competitions popular in the amateur radio community. In a mobile transmitter hunt, a transmitter with a known frequency is hidden. Players then drive around, attempting to locate the transmitter with the use of antennas and other radio equipment.

Our activity takes the form of a video game, in which the player is charged with finding five buried treasure chests that are all transmitting signals from above ground. The player drives a dirtbike in search of each faintly drawn X, indicating where to dig.

Figure 1.1: A faint X indicates buried treasure.

Depending on the terrain, an X might be nearly impossible to make out at a glance, making it difficult to chance upon it.

To help in the quest, the player has an antenna and S meter that can be used together to determine signal direction and strength. The player must switch back and forth between driving mode and scanning mode to mimic drivers pulling over to radio direction find.

Figure 1.2: The S meter gives the signal strength in the direction of the antenna.

If the signal is overwhelming, the player has the ability to attenuate it by listening instead on the third harmonic. This is especially useful when near a transmitter or if several signals combine to max out the S meter.

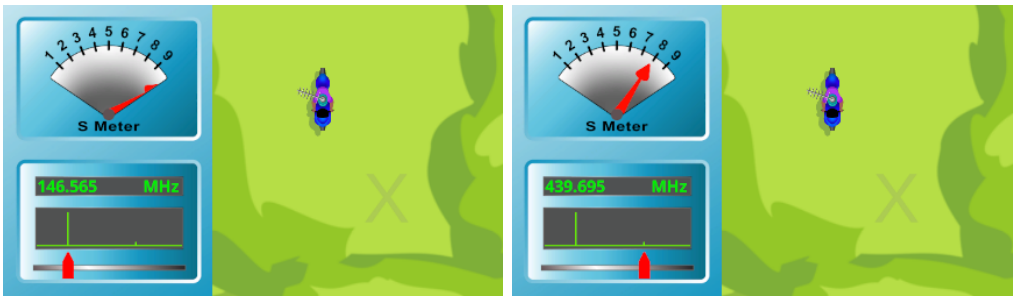


Figure 1.3: Switching to the third harmonic attenuates the signal.

The player is given a minimap of the terrain in the upper left corner. The player's location is shown as a red dot and the player leaves a blue dotted trail as they move around. If the player finds a direction in which the signal is particularly strong, they can draw a ray on the minimap or mark a location of interest with a pink dot. This is to encourage triangulation as a strategy.

Figure 1.4: Example markings on the minimap.

To add to the challenge, the landscape contains forests that attenuate signals and mountains that reflect signals, both of which can be seen on the minimap. A lack of signal in the direction of a forest or a signal in the direction of a mountain must be interpreted by the player.

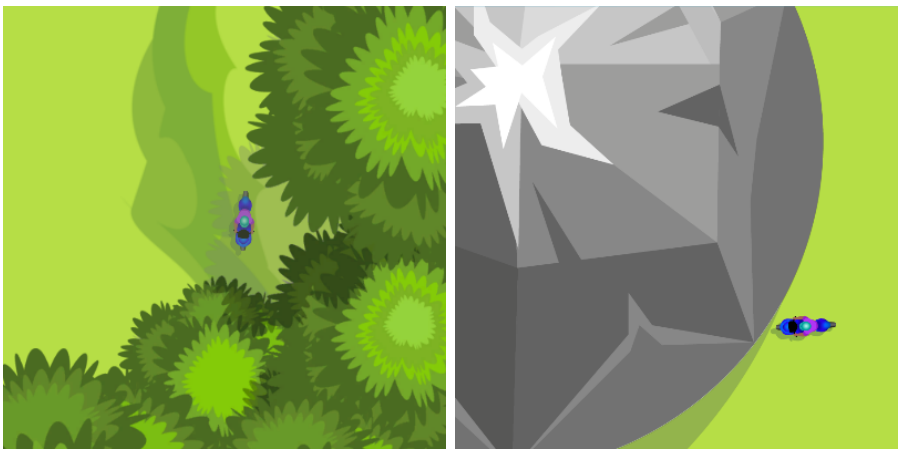


Figure 1.5: Forests and mountains alter signal strength and direction.

On the options screen, the player has the ability to change the number

of treasure chests and introduce new obstacles.

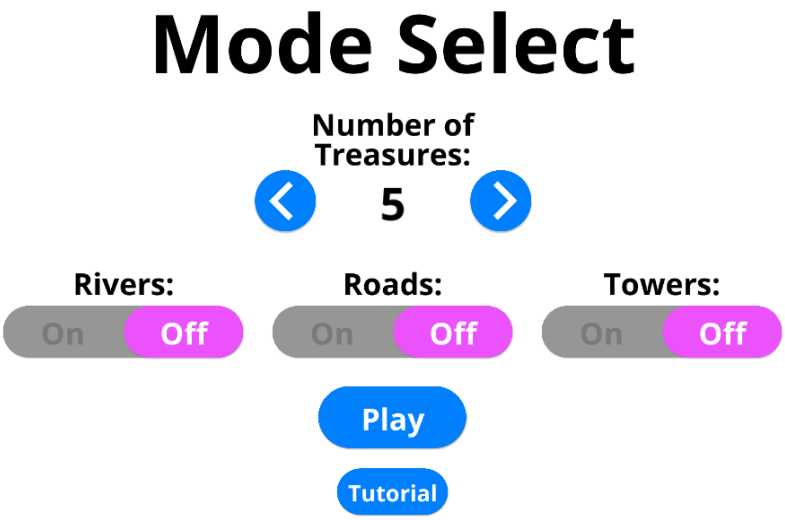


Figure 1.6: The options screen.

Rivers provide barriers that must be crossed at bridges or otherwise traveled around. This encourages the player to break up their search by the regions carved out by rivers.

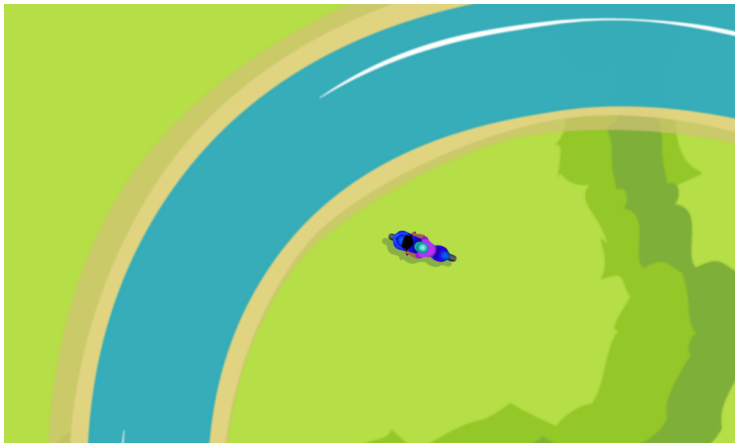


Figure 1.7: A river.

When on a road, the player drives a sporty car and moves quickly. When off of a road, the player moves slowly on foot. This incentivizes certain actions to be taken on the road before embarking on foot.

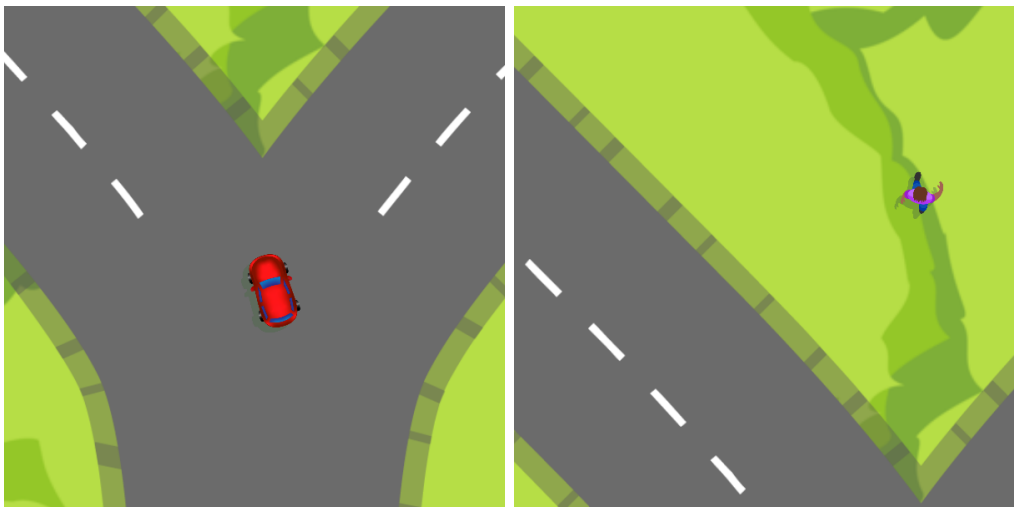


Figure 1.8: A player traveling on- and off-road.

If towers are in play, the player no longer has an antenna except when at one of a handful of radio towers. This is meant to encourage triangulation.

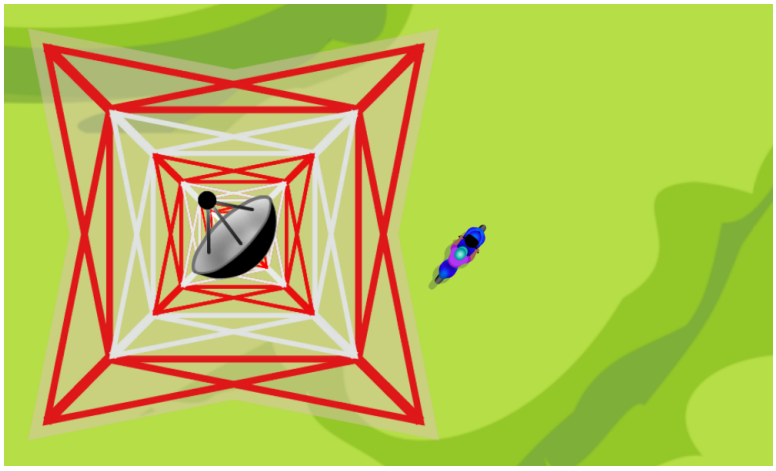


Figure 1.9: A radio tower.

1.2 Technical discussion

Here we present some of the choices made and models used in the backend of the app.

1.2.1 Notation and conventions

In the following, take $\log x = \log_{10} x$.

Generally, we will have a radio transmitting a signal with power P_t and an antenna receiving it with power P_r. Since we will focus on relative strengths as proportions, units won't matter for power and we can adopt whatever normalization is most convenient.

The radio wave has wavelength λ meters and frequency f MHz, with $\lambda f = c$ where $c \approx 3 \times 10^8$ m/s is the speed of light.

The 2-meter (2m) band used by amateurs is the frequency range $144-148$ MHz. Assuming no interference, the standard national (US) transmitter hunt frequency is 146.565 MHz. This occupies a part of the spectrum called very high frequency (VHF), which comprises radio signals in the $30 - 300$ MHz range.

The loss in signal strength is measured in decibels (dB) as

$$L = -10 \log(P_r/P_t).$$

In general, $P_r < P_t$, so L will be a positive value.

When it is necessary to compute P_r/P_t, we will typically first compute L through some formula and then solve

$$\frac{P_r}{P_t} = 10^{-L/10}.$$

Because decibels are a relative unit, it is often necessary to have a baseline measurement, in which case decibel-milliwatts (dBm) are used. The convention here is that a reading of 0 dBm corresponds to 1 mW of received signal power and

$$x \text{ dBm} = 0 \text{ dBm} + x \text{ dB}.$$

A pretty standard transmitter that can transmit over 3 miles in the 2-meter band will have a signal transmission strength of 10-15 mW.

It is often also convenient to work in microvolts (μV). In a system with a 50-ohm (50Ω) load, which is fairly standard in radio, the conversion is roughly

$$x \text{ dBm} + 107 \text{ dB} = y \text{ dB}\mu\text{V}$$

As before, here a reading of 0 dBμV corresponds to a signal of voltage 1 μV.

1.2.2 Signal loss models

If a signal transmitted with power P_t is transmitted a distance d through a vacuum, its expected received signal power P_r is modeled by

$$P_r = \left(\frac{\lambda}{4\pi d}\right)^2 P_t.$$

Rearranging this, we have

$$L_{vacuum} = -10\log(P_r/P_t) = -20\log\left(\frac{\lambda}{4\pi d}\right)$$

For non-vacuums, we refer to "Path-Loss Measurements in a Forested Environment at VHF" by Tan and Stratton for the adjustments to this model that follow. (Their results are adjustments to the standard two-ray ground-reflection model.)

Assuming flat homogeneous terrain ("flat earth"), signal loss is given by

$$L_{ground} = -10\log\left[\left(2\sin\left(\frac{2\pi h_t h_r}{\lambda d}\right)\right)^2\left(\frac{\lambda}{4\pi d}\right)^2\right]$$

for transmitter and receiver heights h_t and h_r meters, respectively. We take $h_t = h_r = 1$, though the model may call for higher placements.

Since $\sin\theta \approx \theta$ for small θ, assuming $d > 10$ we can approximate this as

$$\begin{aligned} L_{ground} &\approx -10\log\left[\left(2\cdot\frac{2\pi}{\lambda d}\right)^2\left(\frac{\lambda}{4\pi d}\right)^2\right] \\ &= 40\log d \end{aligned}$$

Propagation loss through a forest has added attenuation terms:

$$\begin{aligned} L_{forest} &= -10\log\left[\left(2\sin\left(\frac{2\pi h_t h_r}{\lambda d}\right)\right)^2\left(\frac{\lambda}{4\pi d}\right)^2\right] + 10\log(f^{5.4}) - 108 \\ &\approx 40\log d + 54\log f - 108, \end{aligned}$$

In summary, we have the approximations

$$L_{ground} \approx 40\log d$$

and

$$L_{forest} \approx 40\log d + 54\log f - 108.$$

To compute a total loss we can't just sum individual losses, since

$$L_{ground_1+ground_2} = 40\log(d_1+d_2) \neq 40\log d_1 + 40\log d_2 = L_{ground_1} + L_{ground_2},$$

It seems like a reasonable approximation could be to treat any path that passes through a forest as a forest path, even if part of the path is through a clearing, since the forest attenuation term in this model doesn't seem to depend on the distance the signal covers in the forest. The forest attenuation term is

$$L_{forest} = 54\log f - 108.$$

According to this model, we'll run with the loss

$$L_{ground} = 40\log d$$

if the signal path does not pass through a forest and

$$L_{ground} + L_{forest} = 40\log d + 54\log f - 108$$

if it does.

Gound and rock are reflective enough that we can model them as mirrors, reflecting the entire signal. Searching for reflection coefficients of different materials, a formula used for moon bounces is

$$L_{moon} = 10\log\left[\frac{\eta r^2\lambda^2}{64\pi^2 d^4}\right]$$

where $\eta = 0.065$ is the reflection coefficient of the moon's surface, r is the radius of the moon, and d is the distance to the moon. There are more than

a few candidates for extracting familiar terms. Counting the bounce as an event interrupting a single journey of length $2d$:

$$\begin{aligned} L_{moon} &= 10\log\left[\frac{\eta r^2\lambda^2}{64\pi^2 d^4}\right] \\ &= 10\log\left[\left(\frac{\lambda}{4\pi(2d)}\right)^2\left(\frac{\eta r^2}{d^2}\right)\right] \\ &= 20\log\left[\frac{\lambda}{4\pi(2d)}\right] + 10\log\left[\frac{\eta r^2}{d^2}\right] \end{aligned}$$

Counting the bounce as an event that creates a new, directional signal source:

$$\begin{aligned} L_{moon} &= 10\log\left[\frac{\eta r^2\lambda^2}{64\pi^2 d^4}\right] \\ &= 10\log\left[\left(\frac{\lambda}{4\pi d}\right)^4\left(\frac{4\pi r}{\lambda}\right)^2\left(\frac{\eta}{4}\right)\right] \\ &= 40\log\left[\frac{\lambda}{4\pi d}\right] - 20\log\left[\frac{\lambda}{4\pi r}\right] + 10\log\left[\frac{\eta}{4}\right] \end{aligned}$$

So a couple candidates for the bounce loss, isolated from the trip, are

$$L_{bounce} = 10\log\left[\frac{\eta r^2}{d^2}\right]$$

and

$$L_{bounce} = 10\log\left[\frac{\eta}{4}\right] - 20\log\left[\frac{\lambda}{4\pi r}\right]$$

By taking r to be the radius of an object (as a proxy for surface curvature), we can model loss due to a bounce off of a mountain with a moon rock-like material.

1.2.3 Directional antenna

A rough approximation of the shape of the azimuth of a yagi antenna is given by

$$r = (13 - 5\cos\theta + 5\cos 2\theta + 2\cos 3\theta + 5\cos 6\theta)\,(2\cos\theta + 3)\,.$$

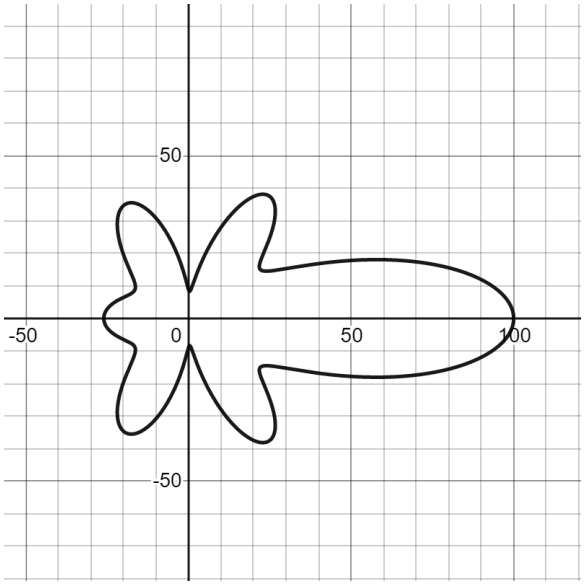


Figure 1.10: Graph of the azimuth approximation.

The maximum radius corresponds to 0 decibels, so if we normalize to

$$L_{antenna} = 40 - \frac{2}{5}\left(13 - 5\cos\theta + 5\cos 2\theta + 2\cos 3\theta + 5\cos 6\theta\right)\left(2\cos\theta + 3\right)$$

we get a formula for signal loss when aiming the antenna an angle θ away from the direction of strongest signal. This allows the app to mimic the local extremum effect transmitter hunters get when aiming their antennas.

1.2.4 Third harmonic

The nth harmonic of a radio wave is a wave in-phase with frequency nf and wavelength λ/n.

According to Nelson and DeMinco's "Improved Estimation of the Third-Order Harmonic Emissions of Land Mobile Radio Base Stations," the received third harmonic power of a signal is roughly 100 dB lower than that of the received fundamental power for free space, line-of-sight transmissions, so

$$L_{harmonic} = 100$$

The third harmonics of the 2m band are the $432 - 444$ MHz range, which falls into the 70cm amateur band comprising $420 - 450$ MHz. These are part

of the ultra high frequency (UHF) range, which is any signal in the $300-3000$ MHz range.

1.2.5 Received signal strength

Here we outline the procedure for computing the received signal strength at the antenna in dBm.

First, each source is considered for its direct line-of-sight paths to the antenna. Each signal will have transmission strength P_i mW (likely in the $10-15$ mW range).

Each signal's transmission power is first converted into dBm using the formula

$$Q_i = 10 \log P_i.$$

Next, the losses suffered by the signal are assessed according to distance covered, whether the signal passes through a forest, antenna angle, and whether the receiver is dialed to the fundamental or a harmonic. In general, we will be using the convention that signal losses (in decibels) are additive along each ray path, so

$$L_i = L_{ground} + L_{forest} + L_{antenna} + L_{harmonic}$$

where we may omit the forest and harmonic terms as necessary.

The received power in mW at the antenna from source i is then

$$P_i' = 10^{(Q_i - L_i)/10}$$

Next, we determine all mountains with line-of-sight paths to the antenna and all transmitters with line-of-sight paths to the mountain. It is then determined whether the mountain has a point on its circumference where the incidence angle and reflection angle are (approximately) equal. If so, this gives a bounce path from transmitter i off mountain j along which the loss can be computed:

$$L_{i,j} = L_{ground} + L_{forest} + L_{antenna} + L_{harmonic} + L_{bounce}$$

Here we take $L_{bounce} = 0$, but may add another attenuation term later, while the distance for computing L_{ground} is the sum of the lengths of the paths from transmitter to mountain and mountain to receiver.

Again, the received power in mW at the antenna from source i along such a bounce path from mountain j would be

$$P'_{i,j} = 10^{(Q_i - L_{i,j})/10}$$

Since thse powers are in mW, they can be summed to find the total power received in mW:

$$P'_{total} = \sum_i P'_i + \sum_{i,j} P'_{i,j}.$$

The reading in dBm at the antenna would then be

$$Q'_{total} = 10 \log P'_{total}.$$

1.2.6 The S meter

The signal strength reading at the antenna is typically interpreted by an S meter. According to the IARU VHF Handbook, for frequencies above 30 MHz the standard calibration of an S meter is that level S9 corresponds to a signal of -93 dBm with a difference of 1 S-unit corresponding to 6 dB. This means

$$n \text{ S-units} = (6n - 147) \text{ dBm}.$$

An S meter typically displays the range S0-S9 in S-units and then higher strengths in increments of +10 or +20 dB after S9 as pictured below:

Figure 1.11: Examples of S meters.

We allow our the S meter to extend a bit past S9, but only so much that the S meter effectively maxes out when close to a transmitter, regardless of the direction the antenna is pointing. This makes switching to the third harmonic necessary to regain directionality.

1.2.7 Background noise

According to ITUR P3728, the background noise can be approximated via

$$F_{am} = c - d \log f.$$

F_{am} is the median background noise measured in decibels above kT_0b, where $k = 1.38 \times 10^{-23}$ J/K (joules per kelvin) is Boltzmann's constant, b is the effective noise power bandwidth of the antenna in Hz, and T_0 is the noise temperature in kelvin. For our purposes, we'll take $b = 5000$ Hz and $T_0 = 290$ K. This gives

$$\begin{aligned} kT_0b &= (1.38 \times 10^{-23} \text{ J/K})(290 \text{ K})(5000 \text{ Hz}) \\ &\approx 2 \times 10^{-14} \text{ mW} \\ &\approx -137 \text{ dBm} \end{aligned}$$

In the formula for F_{am}, f is the frequency in MHz as usual and c and d are constants associated to the environment.

For rural areas, we can take $c = 67.2$ and $d = 27.7$, so

$$F_{am} = 67.2 - 27.7 \log(146.565) \approx 7.2$$

so the median background noise would be -137 dBm$+7.2$ dB $= -129.8$ dBm, or $10^{-129.8/10} \approx 1.05 \times 10^{-13}$ mW.

For quiet rural areas, $c = 53.6$ and $d = 28.6$.

$$F_{am} = 53.6 - 28.6 \log(146.565) \approx -8.3$$

and the median background noise would be -137 dBm-8.3 dB $= -145.3$ dBm, or $10^{-145.3/10} \approx 2.95 \times 10^{-15}$ mW.

Both of these noise levels are detectable at the low end of an S meter.

Using our model for signal loss of a ground wave, since

$$10 \text{ dBm} - 40 * \log(10000) \text{ dB} = -150 \text{ dBm} \approx 1 \times 10^{-15} \text{ mW}$$

a signal transmitting with strength 10 mW with an unobstructed line of sight to the antenna would not be discernible from noise fluctuation in a normal rural environment at 10000 meters. At 20000 meters,

$$10 \text{ dBm} - 40 * \log(20000) \text{ dB} = -150 \text{ dBm} \approx 6.25 \times 10^{-17} \text{ mW}$$

and the signal would not be discernible from noise even in a quiet rural environment.

Since noise is normally distributed, we'll take a normal distribution with mean $\mu = 1 \times 10^{-13}$ mW and standard deviation $\sigma = 5 \times 10^{-15}$ mW.

A transmitter in the 10-15 mW range can usually transmit 3 miles or more. According to our background noise calculations, it seems like an appropriate sense of scale should be that a 10 mW stops being discernable around 10000 meters.

1.3 User response

In addition to informal testing with friends, family, and members of the JRMF community, we tested this activity in a special session of our JRMF community math circle on May 22, 2022. The kids split up into small groups, each led by a facilitator and explored different aspects of the activity based on age, maturity, and facilitator sensibilities. Most participants were in late elementary or early middle school.

1.3.1 Impressions

The main division in users was not mathematical maturity, but rather familiarity with video games. Most who had experience playing video games found the activity intuitive and their primary challenges were the intended challenges of the activity. Those with less experience playing video games had the initial hurdle of learning how to use the interface before taking on the intended challenges. Of those less familiar with video games, the youngest students had the most difficulty catching on.

While most students found the task of finding all of the treasure chests difficult, the problem sparked discussion about good strategies. Some students attempted to find chests by driving around randomly and visually looking for X marks, with little luck. A few students adopted a strategy of driving back and forth, digging up as much ground as they could, hoping to randomly dig up chests. They discovered that the spacing of the holes they dug while driving was rather sparse. Even though they did not need to be directly on top of a chest to dig it up, it seemed there was enough of a gap that their path might pass directly over a treasure chest and still dig up the treasure:

Figure 1.12: Hole spacing made by digging while driving.

These strategies did not work, so students tended to evolve some combination of three strategies:

1. Drive randomly, stopping periodically to take radio reading. When a signal is detected, drive in its direction, stopping occasionally to take another reading to course-correct until an X is located.

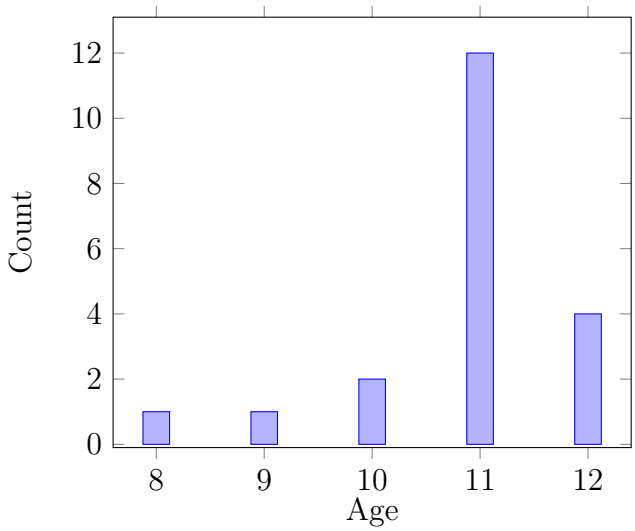
2. Drive randomly, stopping periodically to take radio reading. When a signal is detected, mark the direction on the map with a ray. Move away from the ray and take another measurement, again marking the direction with a ray. If the rays intersect, head to the intersection and visually search for an X.

3. Break the map up into quarters, sixths, ninths, or some other sectoring. Search each sector one at a time, stopping to take measurements and looking for X's.

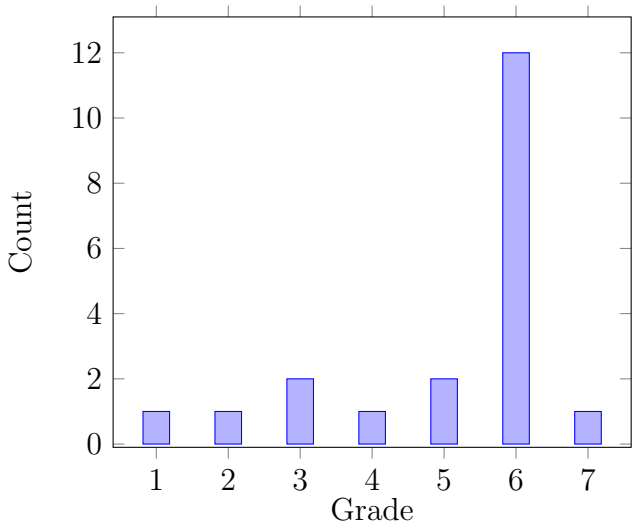
The first of these strategies was the one most students stumbled upon and tended to work pretty well in the default mode. The sector strategy was not super popular but had a couple vocal proponents. Most of the students who triangulated learned to do it while exploring a mode with radio towers enabled and then used it as a strategy even when playing without towers.

1.3.2 Survey results

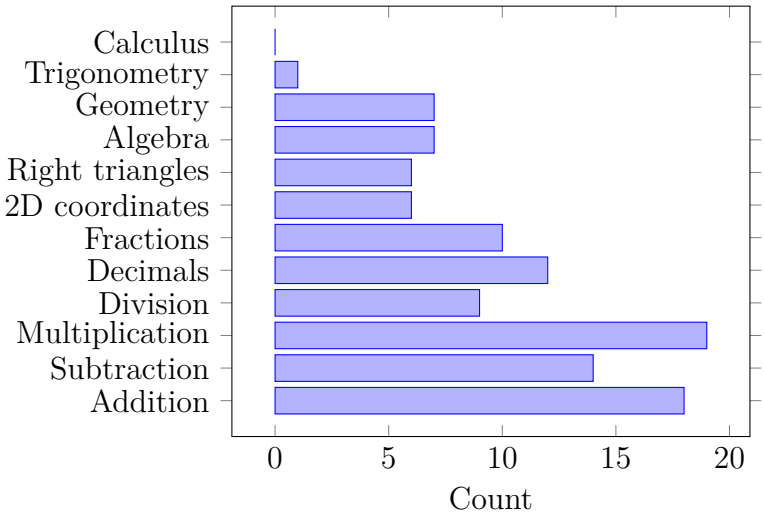
1. What is your age?



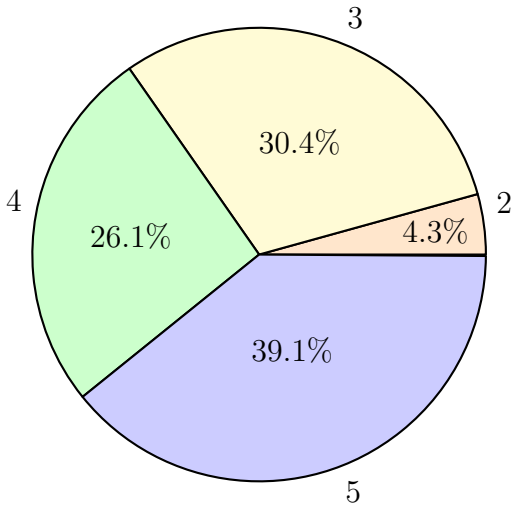
2. If currently in school, what is your current grade or year?



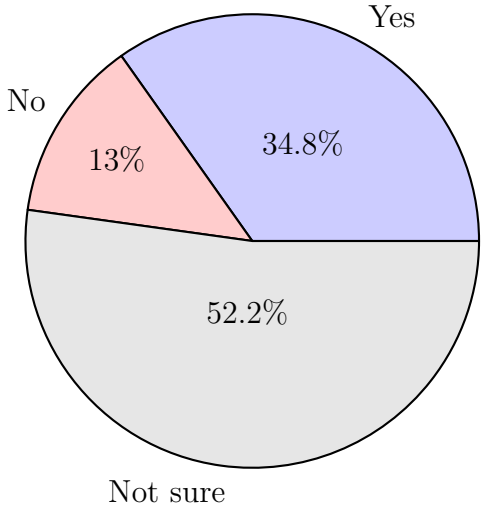
3. Which of these math topics are you somewhat comfortable with? (check all that apply)



4. How much did you enjoy the activity? Scale: 1 (I did not like it at all) to 5 (I liked it a lot)



5. This activity was based on an activity people do outside with real radios. Do you think that is something you might like to try?



6. What did you like about the activity?

it was fun
fun
easy
money
you find chests
i like that you get to drive around and find things and change the setting
idk
the finding part
it was fun like dogee miner
the math
its a very well put together game. i love driving around fast
the game
i got to drive a bike
How you could drive until you found the signal
I can see some students loving this!
the different signals pointing n different directions
I liked moving around searching for it (Treasure)
It was really fun when I suddenly found the treasure.
fun to do and fun to learn
I like that it was interactive and that there were different modes.
Well, Lots of things! but one was the fun! :D

7. What didn't you like about the activity?

nothing
time
buggy
finding the treasers
it was hard to find the chest
that it was hard to find the tresure
nutin
idkv
BUSHES
?
the x was hard to find
how hard the game was
boring kind of but fu
You couldn't use the signal finder when you were in the road-man-car.
Took some time to find a chest!
i don't know
Nothing
It was a bit hard to find the treasures
lots of different approaches
I'm not a big fan of video games. Also, I didn't really get any geometry-or-so strategies.
Well, there was a bug but it was very minor...
boring

8. What did you learn from the activity?

idk
treasure
radio waves
never give up
to explore
how to tell the signal
how to find raido signils
never give up
tracking and cordnents
how to work a raidio
nothing, but i had fun
nothing
You have to use the signal finder instead of just driving up then move down
Frequency
about radioing for treasure
Graphing, maybe and the frequencies.
you can lower or higher the frequency to find it.
about frequencies
I learned how to predict the place.
What Frequencies Are!
how to find a thing with radio

9. Do you have any suggestions for improving the activity?

no
not really
turn into boat
better controls
make the controls easeare to movce
bigger screen to see around you
make the x‘s stand out more
notjhing
customizing your car and being abel to open the chest to unlock more cars
make the x more seeable
add more locations to do such as boat and airplane and add animals that chase you
the one you showed us
No
NA
No, it was really nice
easier to use with mouse so it does not go so fast
No, not really.
Remove The Bug(s)

Chapter 2

Resistors

2.1 Activity overview

Resistors is an experimental activity intended to give the user the feeling of conducting a scientific inquiry into the effect resistance has on current and voltage in an electrical circuit. We wanted it to feel like an exploratory mathematical activity, where students encounter a phenomenon, perform calculations, make conjectures, and decide whether their conjecture still holds. Since the topic is scientific instead of mathematical, the app plays the role of the natural world and a series of puzzles introduce increasingly complex scenarios intended to ease students from one discovery to the next.

The first set of puzzles consists of circuits with ideal current sources. The circuits feature gaps in different parallel and series arrangements and ammeters placed at different points, allowing the user to read the current. Each ammeter has a target current and the puzzle is solved when resistors have been placed in such a way that all target currents are realized.

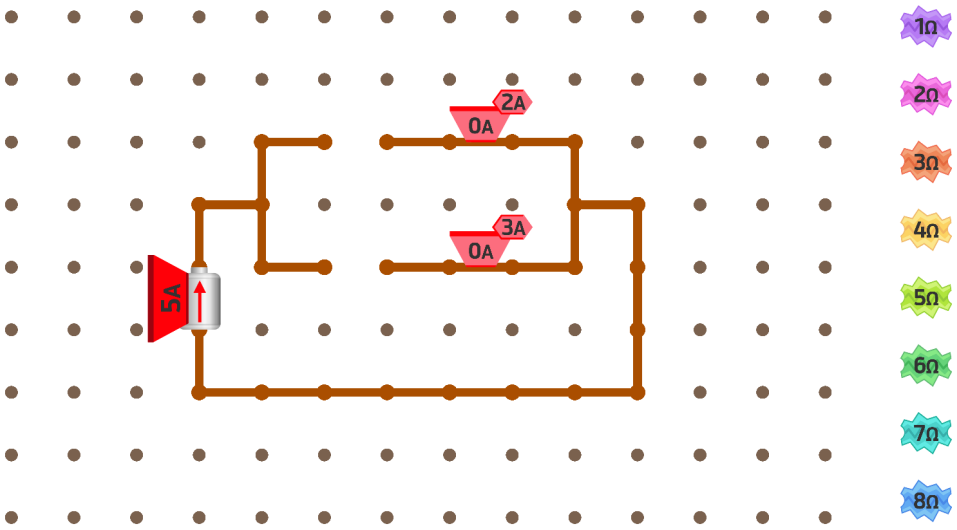


Figure 2.1: An ideal current puzzle.

The solutions to the puzzles are not necessarily unique. Finding several solutions to a puzzle is encouraged, since this is useful for formulating, confirming, and rejecting hypotheses. It also helps students avoid the trap of guessing a solution and accidentally bypassing a lesson they were intended

to learn for later puzzles.

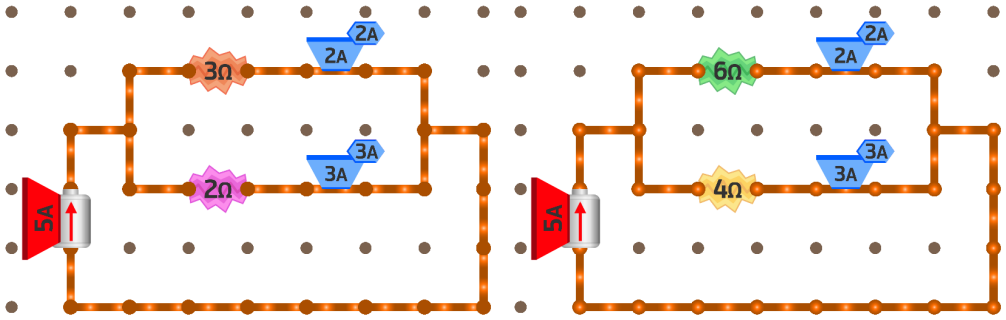


Figure 2.2: Two solutions to the same current puzzle.

Students are still given feedback in the form of ammeter readings for incorrect solutions, giving them a sense for how the resistors work together. This allows them to make imprecise observations as stepping stones to more precise observations. For example, "higher resistance seems to allow less current through" will be apparent before "resistance and current are inversely proportional."

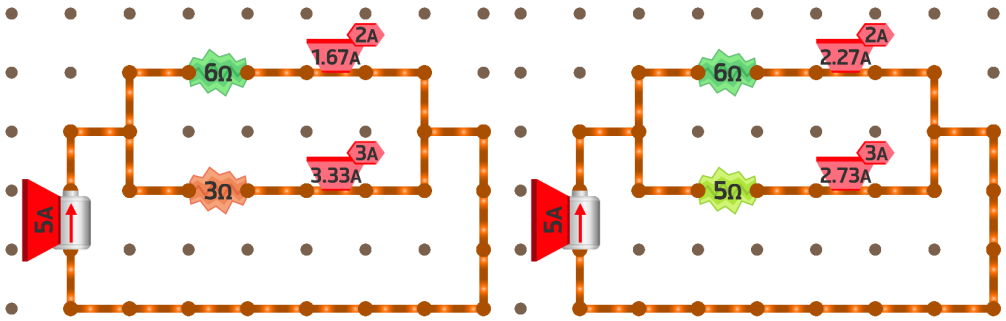


Figure 2.3: Two non-solutions to the same current puzzle.

The puzzles steer students to make conjectures about

- Current in closed versus open circuits
- The ways that current is conserved

- Behavior of current over resistors placed in parallel
- Behavior of current over resistors placed in series
- The topology of circuits

The circuits get more complicated in order to focus on features where something matters that might not have mattered before. Since voltage does not make an appearance, the focus is intended to be on simple arithmetic relationships amongst currents and resistances in the form of products and ratios.

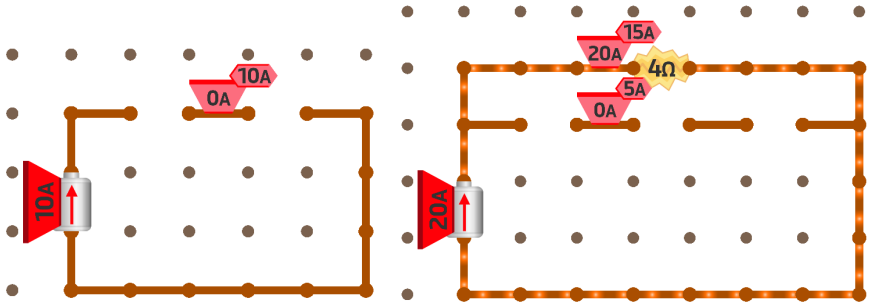


Figure 2.4: An early series puzzle and a later series puzzle.

A second sequence of puzzles introduces an ideal voltage source and has students explore the relationship between resistance and voltage.

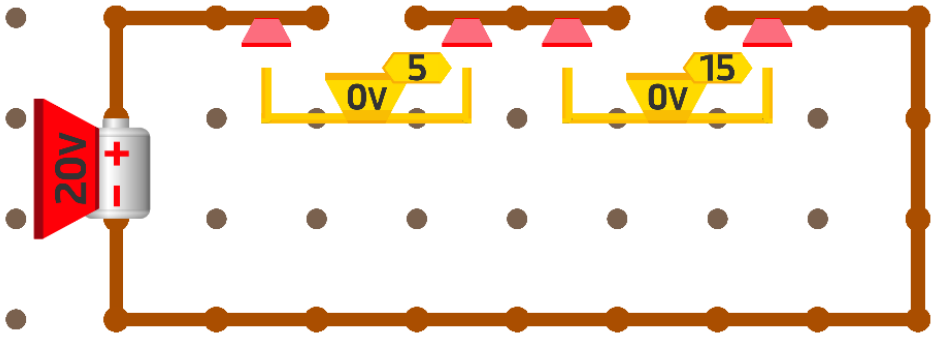


Figure 2.5: An ideal voltage puzzle.

As with the previous sequence, most puzzles do not feature a unique solution.

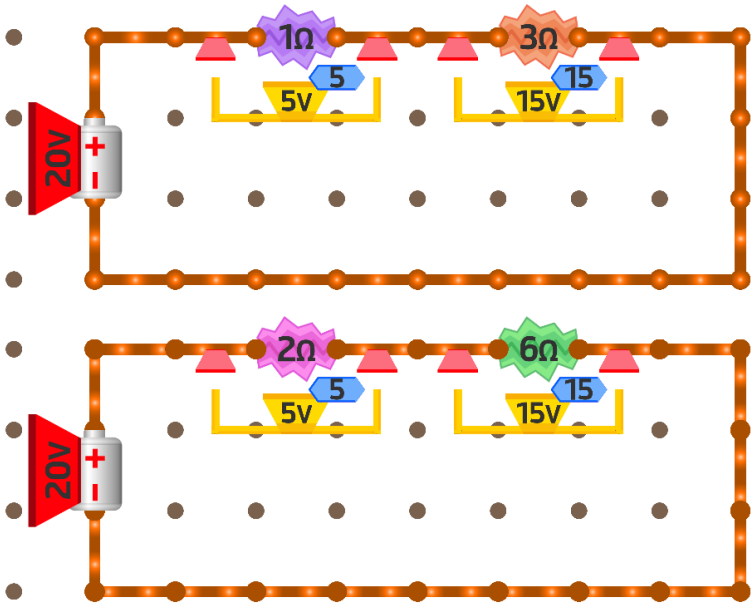


Figure 2.6: Two solutions to the same voltage puzzle.

Students are encouraged to develop their understanding of the interplay between resistance and voltage in increasingly complex circuits, focusing mainly on how voltage is affected by placing resistors in different series and parallel configurations.

2.2 Technical discussion

Here we present some of the science and the philosophies behind how it was adapted for use in the app.

2.2.1 Circuit laws

If V represents voltage, I current, and R resistance, then the main laws that students learn in a course dealing with circuit analysis are

- Kirchhoff's current law: At any node, if $I_1, \ldots, I_n$ are the currents in all branches adjacent to the node signed positive if flowing in and negative if flowing out, then
$$\sum_{k=1}^{n} I_k = 0$$

- Kirchhoff's voltage law: In any closed loop, if $V_1, \ldots, V_n$ are the signed voltages around the loop, then
$$\sum_{k=1}^{n} V_k = 0$$

- Ohm's law: In a segment of wire, the current flowing from one end to the other is proportional to the voltage across it, where the constant of proportionality is the resistance
$$V = IR$$

These lead to a couple basic observations:

- If a circuit forks into several branches, then the total voltage is the same in each branch while the sum of the currents in each branch equals the total current in and out.

- If several sections of a circuit are in series, then current is the same in each section while the sum of the voltages across each section equals the total voltage from start to end.

2.2.2 Products and ratios

If a circuit forks into two branches, then we know that the two branches have the same voltage V, while the currents I_1, I_2 and resistances R_1, R_2 may vary. In this case, Ohm's law gives

$$I_1 R_1 = V = I_2 R_2.$$

In each branch of a parallel circuit, we must then have $I_1 R_1 = I_2 R_2$. If we wish to achieve target currents, we can then think about placing resistors so that the IR products equal in the branches, or find resistors R_1 and R_2 so that $R_1/R_2 = I_1/I_2$. With more than two branches, this is most easily thought about in terms of products, though it can be conceptualized in terms of finding a vector $(R_1, R_2, \ldots, R_n)$ parallel to $(1/I_1, 1/I_2, \ldots, 1/I_n)$. This provides a mechanism for focusing on the inverse relationship between resistance and current without needing to invoke voltage.

Similarly, if two sections of a circuit are in series, then the same current I must flow through each section. As for the voltages and resistances in each section, they must satisfy

$$V_1/R_1 = I = V_2/R_2$$

This allows us to focus on the proportionality of voltage and resistance without needing to invoke current.

The hope is that this focus on two quantities at a time rather than all three makes it easier to divine the relationships.

2.2.3 Diophantine problems

Many of the puzzles have multiple solutions. While the early puzzles can be solved using intuition, later puzzles will require making more explicit conjectures.

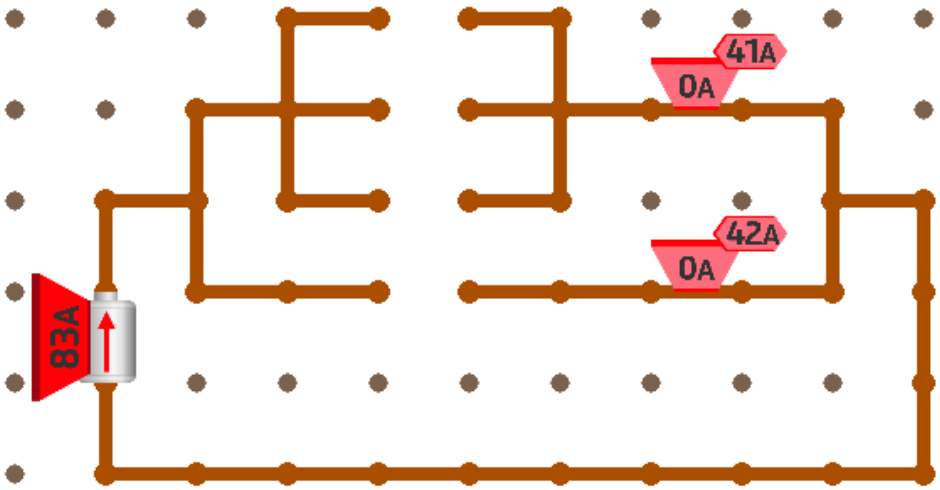


Figure 2.7: A late current puzzle.

In the puzzle above, if R_1, R_2, R_3, R_4 are resistances that solve the puzzle, filling the slots from top to bottom, then

$$41\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)^{-1} = 42R_4,$$

giving

$$41\left(\frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_3R_1}\right) = 42R_4$$

or

$$41R_1R_2R_3 = 42R_4(R_1R_2+R_2R_3+R_3R_1)$$

Since 41 is prime and all resistances are in the range $1-8$, this is a sign that we probably need $42R_4$ to divide $R_1R_2R_3$. Since $42 = 2\cdot 3\cdot 7$, we can try $2, 3, 7$ for R_1, R_2, R_3 and $R_4 = 1$. This is indeed a solution!

Since the formulas we write down have this sort of Diophantine character for a limited domain, these sorts of divisibility arguments can help solve later puzzles.

2.3 User response

In addition to informal testing with friends, family, and members of the JRMF community, we tested this activity with two Vancouver, BC high school computer science classes on June 14, 2022. The students were introduced to the activity and then worked in pairs to solve the puzzles. A facilitator would pop around and ask them about their discoveries and give them hints when they were stuck.

2.3.1 Impressions

The range of skill and experience of the students was pretty wide. Most were very comfortable with algebra and ratios, which was the important factor.

Several had taken introductory physics courses and indicated that they thought the activity was based on Ohm's law. Although that could possibly be a boon, it seemed to mostly rut those students into thinking about problems in a particular way and stymy them when they could not apply a formula.

On the other hand, students with less experience in physics seemed to have a better shot at recognizing some of the relationships between current and resistance based on products and ratios alone, which was part of our goal. Some arrived there through hints, but about many discovered the relations entirely on their own.

Some of the students discovered that the interface allowed them to place resistors in unintended places. Some made a game of this, but others used it to create creative, though unintended, solutions.

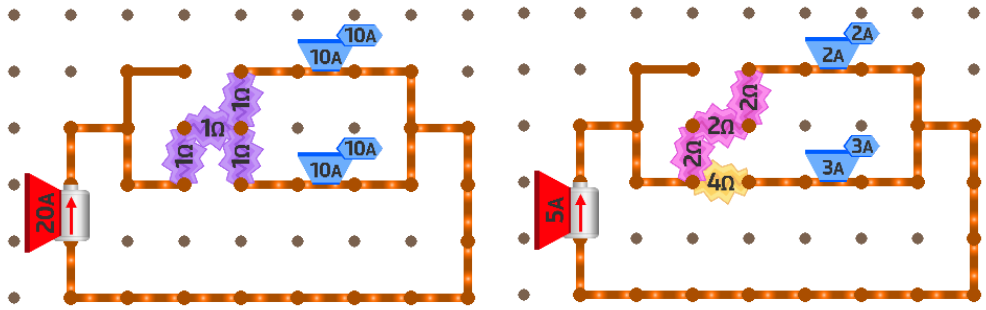


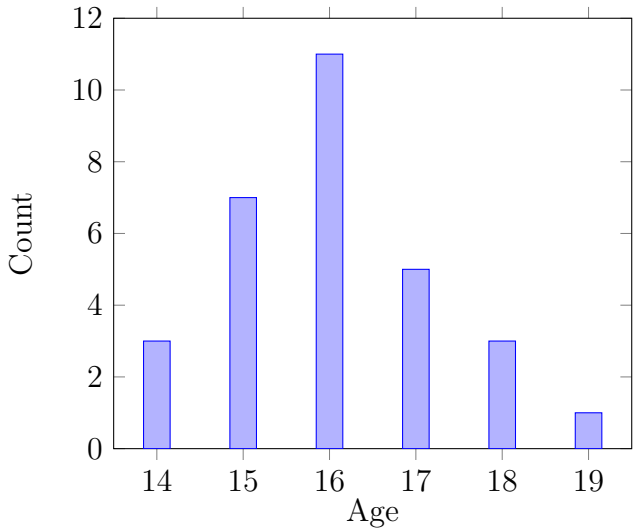
Figure 2.8: Unintended solutions to puzzles.

This sparked a discussion about whether to limit the places resistors can be placed. These creative solutions were not particularly useful in the lesson arc, since the puzzles were sequenced to allow particular observations to be made and built upon. However, they did get students wondering about why their solutions worked, despite the reasoning required being something they were intended to grapple with later. The sentiment was split, so we ended up not disallowing these.

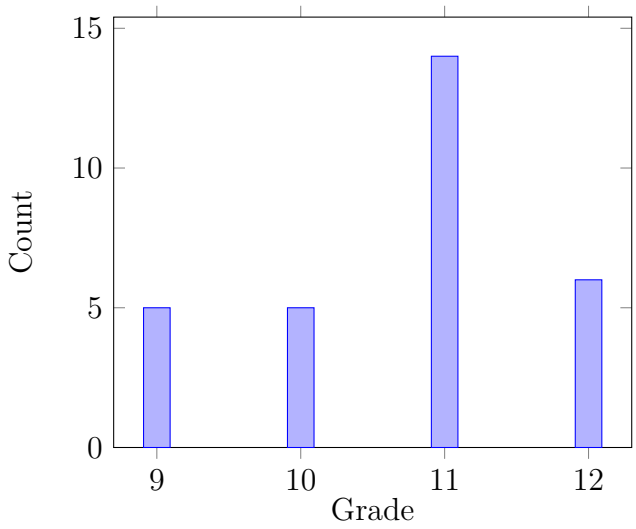
The main point of frustration was around a lack of built-in explanations and hints. This was a design choice on our part, so it's not clear if the activity requires redesign or if it hasn't met with its intended audience. The students who made good progress were very proud of their discoveries, mirroring what we tend to see in a good math exploration. Regardless of any reported frustrations, only 6.7% rated their experience unfavorably while 60% rated it favorably.

2.3.2 Survey results

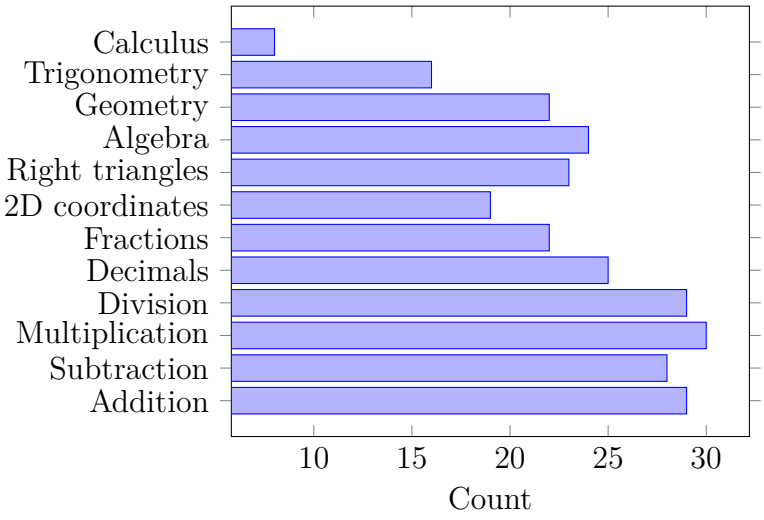
1. What is your age?



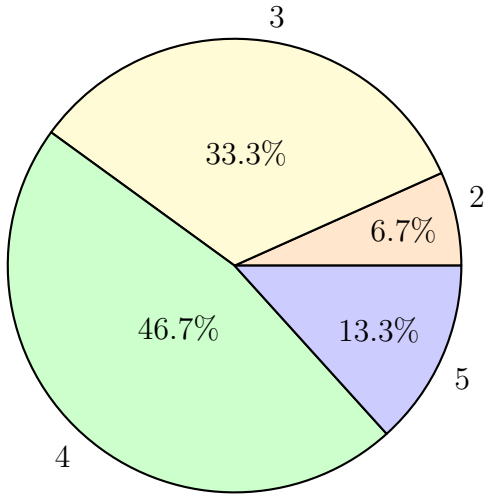
2. If currently in school, what is your current grade or year?



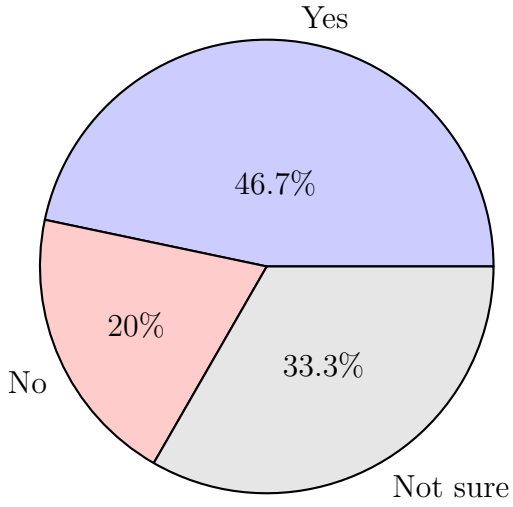
3. Which of these math topics are you somewhat comfortable with? (check all that apply)



4. How much did you enjoy the activity? Scale: 1 (I did not like it at all) to 5 (I liked it a lot)



5. This activity was based on some principles of electronic circuit design, which people do both for fun and for work. Do you think that is something you might like to try?



6. What did you like about the activity?

Discovering how Ohm's law works.
problem solving
Interesting
hands-on/interactive
it was fun trying the different numbers
The colours and presentation were visually appealing and creating new solutions made me use my brain.
It showed us the amps flowing through the circuit
The interactive puzzles.
Finally getting the puzzle right after getting decimal answers
puzzle
I liked being able to come up with patterns that seems consistent with the same types of problem.
it was educational
It was visually appealing
You can figure things out in many, many ways
It was completely hands-on, and the concept is a great idea
I liked learning more about circuitry
I like that it was left for the students to solve and make their own conclusions.
The software
I liked doing the activity online and the website
I like experimenting trying to figure out patterns
I liked thinking what my next move should be (calculating the voltage and resistance)
I don't know
puzzles
It's interactive and helpful for people who would have to study DC circuits. It's not to complicated and the problems allow you to really think instead of just guessing.
Resistors logic
How simple it is to understand.
it was cool

Got harder as you progress each level, could backtrack on previous levels.
the custom mode
it is something new i havent done

7. What didn't you like about the activity?

I would've liked it if there was a little bit more context about how amps and resistors work.
dragging the numbers in and out was annoying
Website was buggy sometimes
Couldn't finish all the activites
I didn't really understood how it worked
Some of the solutions created by other people worked, but they didn't connect all the wires. (bugs)
I was stuck on question 16.
The puzzles were hard.
Not getting the puzzle right
brain hurt
I didn't like how I couldn't really use standard physics formulas like V = IR for the current activity, as I was trying to find each branch's current via math. Though, that isn't really an issue for this particular activity.
there was little to no instruction
There is no point to doing it. After the first couple puzzles it was very repetitive and not rewarding.
It took you a whle to understand how the activity work.
There wasn't much instruction from the site
I didn't like that there wasn't a broader discussion of the topic
There was a lack of guidance on what we were learning sometimes.
I didn't know how to solve the problems without guessing
i have no dislikes
No hints on a mathematical solution, since i was trying to figure out a mathematical formula i could use to get a reliable answer.
It does seem a bit linear
I don't like math that much.

no sounds
There weren't anything like hints. For the really hard levels that would've been helpful.
tutorials
It seems like it lags the computer if you add too many resistors
i didn't like how it was just a bunch of guessing
Didn't understand how it worked, no way I could find to return to tutorial when skipped.
how there was no explanation to the activity
it was confusing

8. What did you learn from the activity?

circuits
Nothing
resistors divide current
ratios of the ammeter
Resistors
Relationships between voltage, input, and resistance
there are mathematical aspects to consider
How to create correct links and measures for the ammeter.
Amps + Resistors
How circuits work.
I learned more about resistors and was able to reuse a little bit of what i learned from physics
I re-learned how series and parallel circuits worked.
i learned most of the concepts from physics 11
Nothing really
I learn more about how resistor work, how parralel and series current are different.
I learned that I forgot how certain circuits work with resistors (as I've learned of resistors already in Physics 11 and 12)
I learned the effects certain amounts of ohms have with resistors

I learned about voltage and how amps are changed/reduced with resistors.
Nothing really
I learned about electrical circuits
Resistance is weird
How to calculate the required resistors given the total current and specific ammeters
I learnt a little more about how ammeters and resistors work.
How to calculate resisstors
How to calculate resistance with resistors.
nothing
battery/ evening them out

9. Do you have any suggestions for improving the activity?

No
Tell us what current and resistors do specifically and how to gain a better understanding.
no, it is well made
Website needs more instructions
More instructions
I found some bugs, the measure thing would show "NaNa" but it said that i did it
Adding hints would be nice
sound design !!!!
It's not really something major, but I would like the Voltage of the battery in the current section to be listed for those who like to solve it through math.
give more instruction
Give some kind of an overarching goal
You need a base knowledge about how electricity work and for newbie i think you can add a button for hints so people can figure things out easier.

- Is it possible that a resistor can be swapped with another on the board by dragging one over another? I found it slightly annoying to change resistors as it required me to move the one originally away first - Maybe add some hints/tips, or an explanation on how the circuit works on level completion. Like "The current on the top path is four amps because of the ten amps of the initial current, the resistors made it so only 2/5 of the current passes through the top path" or something like that.
Talk about turning DC motors to generate power
It would have been nice if the process was more step by step.
Tell us how to do things
no
Hints are always nice to push people into the right direction, however they take like way too much work
You could have a toggle that shows the voltage when dealing with the currents. That would've helped me figure out what was happening. Make it a toggle because just showing the voltage from the beginning would be too easy.
Nope
sound design
The overall design of the game is perfect but I would definitely try to add something like hints in the levels, even if it's a really small hint.
none
Make the placement have rotate buttons instead of auto placement.
A better tutorial i guess so people could understand some basics
Add a better tutorial system. Make resistors easier to place when placing on dots.
Hints for if the player is stuck
more clear possibly/easier to understand

Chapter 3

Cryptography

3.1 Activity overview

Cryptography is an activity that extends two traditional math enrichment activities:

- Learning about cryptography via Caesar ciphers and cipher wheels.
- Solving cryptograms based on simple alphabetic substitution ciphers.

These two activities are related in the sense that Caesar ciphers are a subset of simple alphabetic substitution ciphers.

The first sequence is on Caesar ciphers. In each puzzle, the player is given a poem encrypted using an unknown shift parameter. The player must determine the shift parameter and decrypt the poem.

The use of poetry was inspired by the Poetry in Motion campaign, via which the poetry from a diverse authorship was displayed on public transit in various cities throughout the United States. Many of the poems have been borrowed from that campaign, with a preference for those about animals and nature.

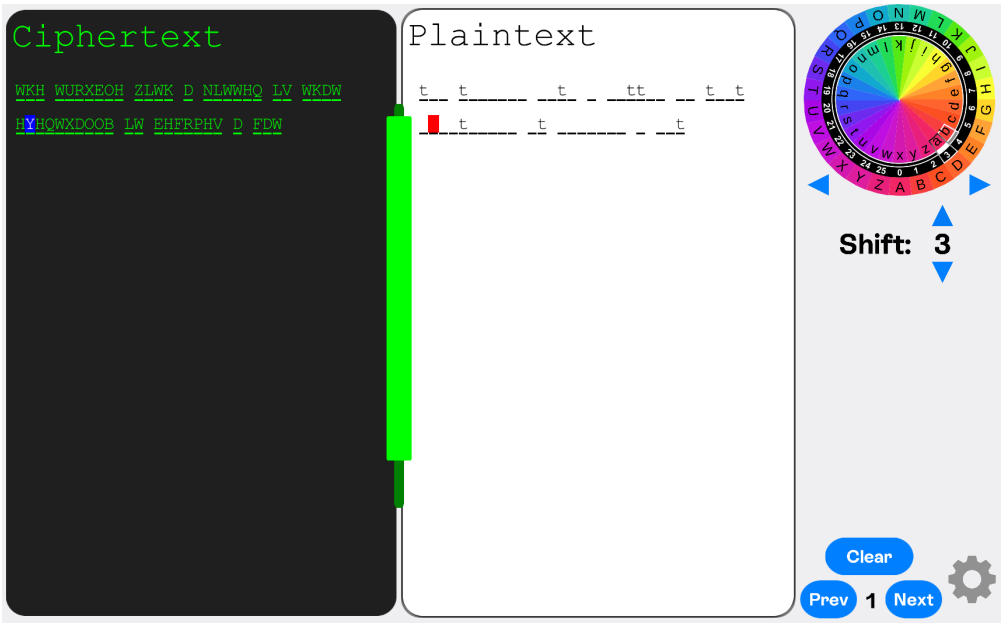


Figure 3.1: An encrypted poem with one letter entered.

By default, the player edits all instances of a single letter at the same time. There is an autofill option that will populate the plaintext field based on the selected parameters, but the player is not intended to discover this until they have worked on a few puzzles. Once a poem has been solved, the title and author appear at the top with a footnote about the collection it appears in, if available.

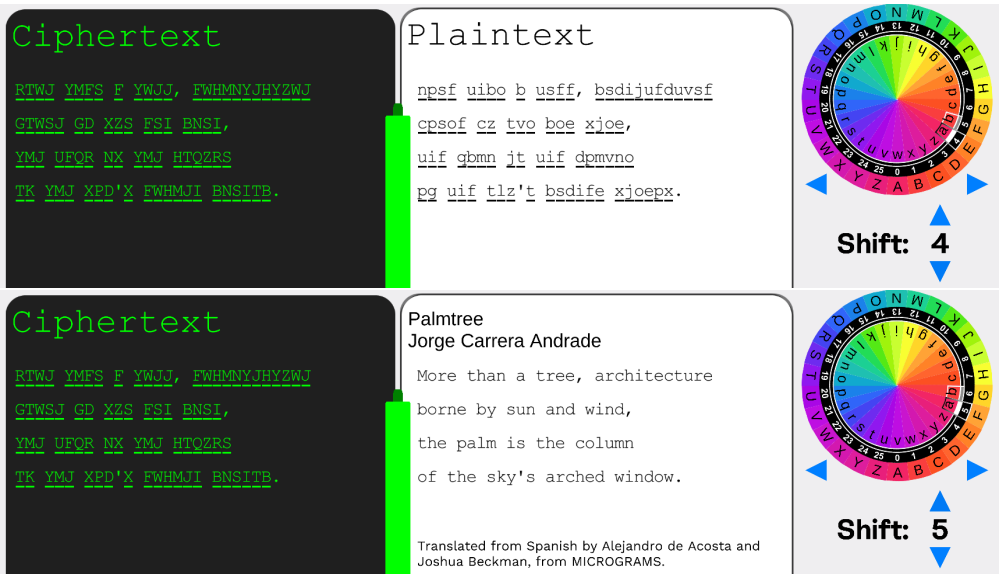


Figure 3.2: A poem solved by changing the shift parameter with autofill enabled.

To enable autofill, the user must enter the options menu by clicking the gear icon, click the lock icon, type in the password “ebg”, and hit enter, and drag the switch from "Off" to "Auto." (EBG is ROT encrypted with a shift of 13.)

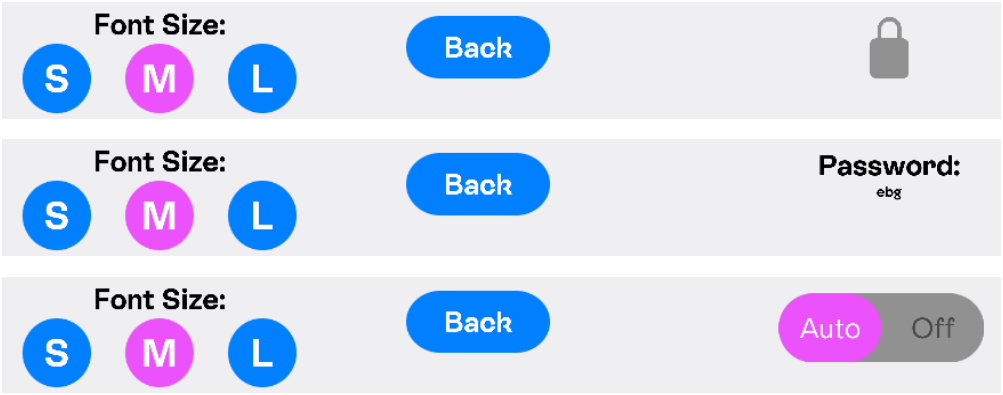


Figure 3.3: Unlocking the autofill option.

The other sequences are also accessed from the options menu and deal with related ciphers:

- Affine ciphers
- Vigenère ciphers
- Simple alphabetic substitution ciphers

Affine ciphers present an increase in difficulty from Caesar ciphers as direct generalizations, while Vigenère and substitution ciphers present increases in difficulty in different directions. Since the later ciphers require more contextual clues than the earlier ones, the poems increase in length as the player progresses through the modes.

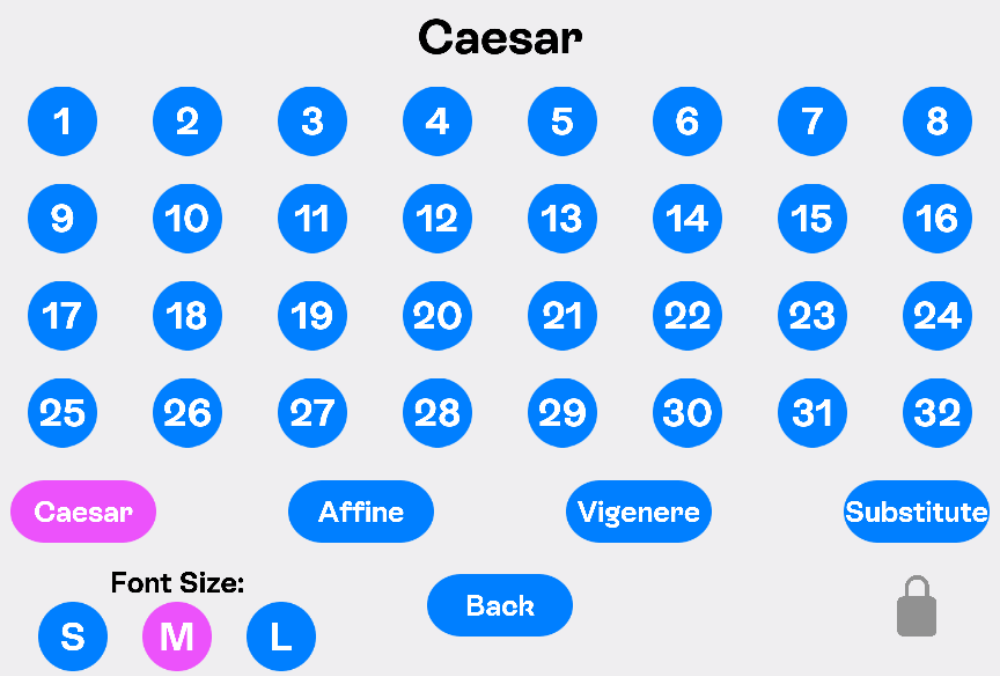


Figure 3.4: The other puzzle sequences in the options menu.

3.2 Technical discussion

The encryption schemes employed are all types of substitution ciphers. Here we discuss these ciphers and some of the design choices made.

3.2.1 Affine ciphers

The most common cipher students first explore is the Caesar (shift) cipher, where each letter is translated a set distance down the alphabet, wrapping around at the ends. Since these ciphers are often implemented by writing the alphabet on two concentric disks and rotating them relative to one another, they are often described in terms of rotations instead of shifts.

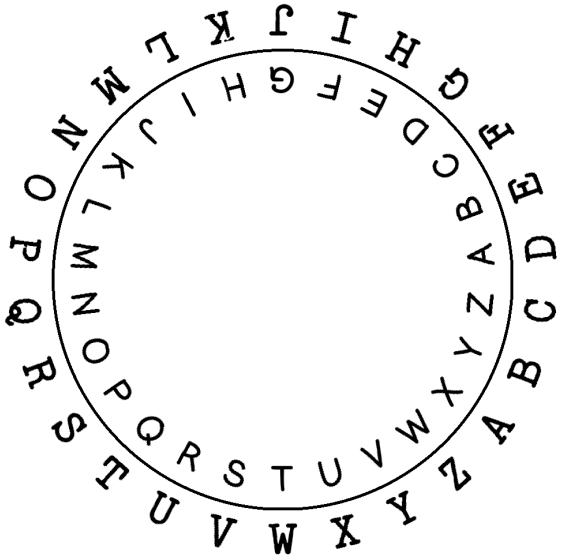


Figure 3.5: A Caesar cipher wheel with a shift of 3.

In the figure above, reading inside to outside, we see that A is encrypted as D, B as E, and so on. Similarly, reading outside to inside provides a key for decryption. As you can see, directionality (in-to-out vs out-to-in) give two different schemes: while A encrypts as D, A decrypts to X.

Another common cipher, colloquially called the Atbash cipher, comes from reversing the alphabet. If we're willing to reverse the orientation of one of our wheels, we arrive at a key:

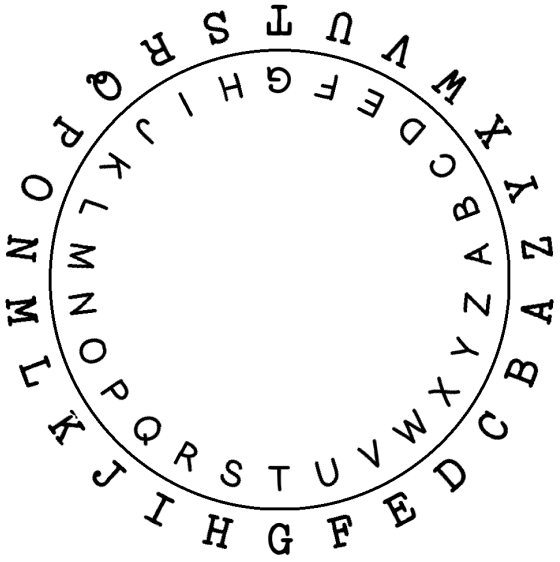


Figure 3.6: Atbash cipher key.

For this one, directionality is less important, since encryption is the same as decryption.

Arithmetically, we can describe a Caesar cipher as follows:

1. Convert the letters of the alphabet to numbers $0 - 25$
2. Add the shift value to each number, wrapping to keep values in the range $0 - 25$
3. Convert the numbers back to letters

If d is the numerical value of an unencrypted letter, e is the numerical value of the encrypted letter, and s is the shift constant, then this is

$$e = (d + s) \mod 26$$

where $x \mod n$ takes x and returns the unique value y so that $x - y$ is divisible by n and $y \in \{0, 1, 2, \ldots, n - 1\}$. Decryption is done via

$$d = (e - s) \mod 26$$

The Atbash cipher has a similar arithmetic formula:

$$e = (25 - d) \mod 26$$

Decrypting is identical:

$$d = (25 - e) \mod 26$$

More generally, Caesar ciphers and the Atbash cipher are part of a family of ciphers called affine ciphers. An affine cipher has the form

$$e = (m \cdot d + s) \mod 26,$$

where s is a shift factor and m is a slope or spacing factor. While s can be any value in $\{0, 1, 2, \ldots, 25\}$, for the encryption scheme to be decryptable, we require that m and 26 be relatively prime. In effect, we can only take $m \in \{1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25\}$.

Decrypting these is a little more subtle. While division is not well defined (mod 26), by choosing m relatively prime to 26, we ensure that a multiplicative inverse m^{-1} exists with

$$(m^{-1} \cdot m) \mod 26 = (m \cdot m^{-1}) \mod 26 = 1.$$

Here is a table of multiplicative inverses:

m	m^{-1}
1	1
3	9
5	21
7	15
9	3
11	19
15	7
17	23
19	11
21	5
23	17
25	25

Figure 3.7: Multiplicative inverses (mod 26).

Using this table, we have a decryption formula for the affine cipher $e = (m \cdot d + s) \mod 26$:

$$d = m^{-1} \cdot (e - s) \mod 26,$$

3.2.2 Crypto wheels

While any Caesar cipher can be implemented using a single wheel simply by rotating the inner disk for each shift factor, there is no similar mechanism for affine ciphers, other than creating 12 different wheels – one for each value of m. Since this is an app, however, the wheels don't need to be physical objects, giving us some leeway in creating alternative configurations. Since the parameter m can be though of as creating spacings in the domain or codomain, wheels give nice geometric interpretations of these arithmetic parameters. To make this spacing phenomenon more visually striking, we used a color wheel to color the alphabet, with A to Z forming a continuous color spectrum.

For a standard Caesar cipher, you can see the shift constant in terms of a color shift. In the example below, the $s = 3$ wheel looks less drastically shifted than the $s = 13$ wheel, since like colors are near each other:

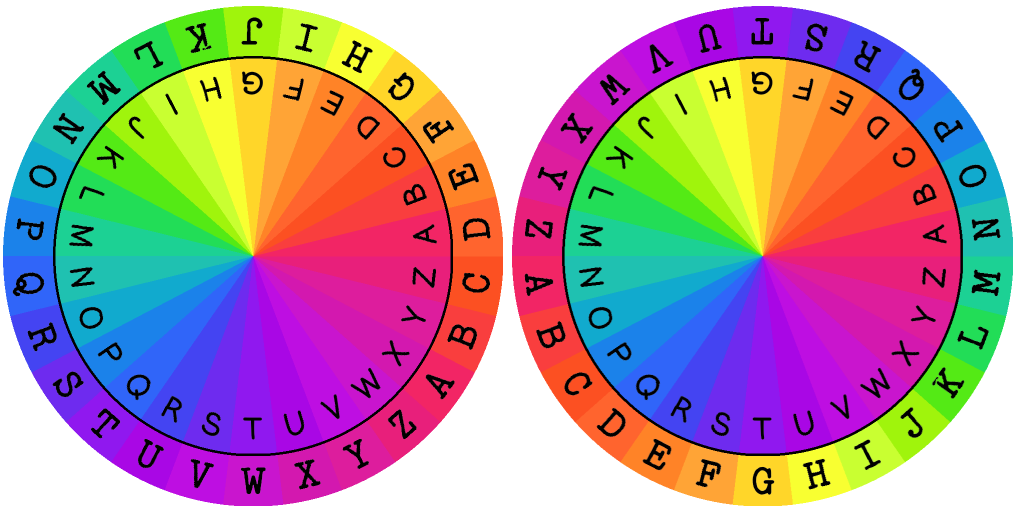


Figure 3.8: Crypto wheels for Caesar ciphers $s = 3$ and $s = 13$.

For the Atbash cipher, opposite orientations of the wheels is more visually

distinguishable, since it is clear that the rainbow ordering is reversed from the inner wheel to the outer wheel:

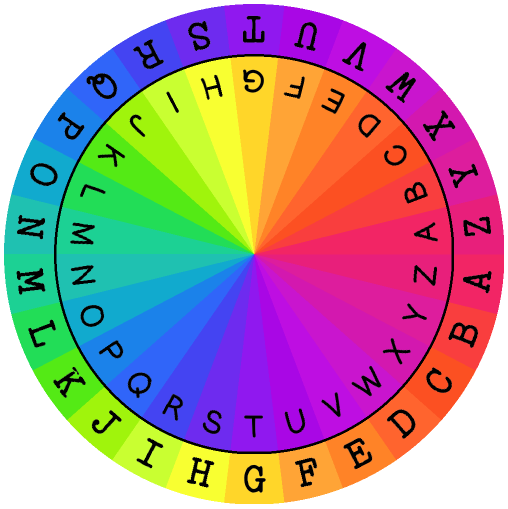


Figure 3.9: Crypto wheel for Atbash cipher.

While this distinction doesn't matter for the previous ciphers, for a given affine cipher there are two nicest ways to present the wheel: keep the inside wheel alphabetical and rearrange the outside or keep the outside alphabetical and rearrange the inside. If we adhere to the convention of encryption inside-out and decryption outside-in, then the first way is more convenient for encryption and the second way for decryption.

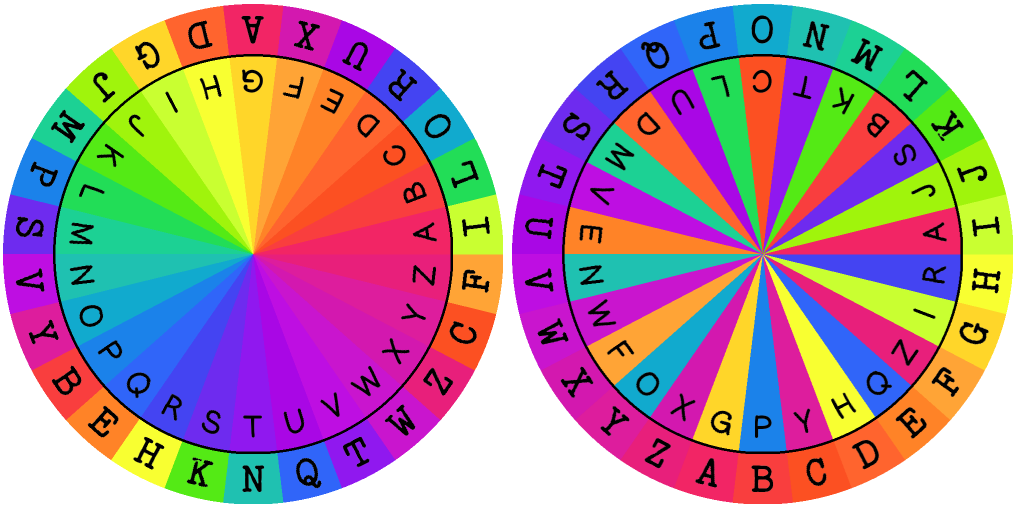


Figure 3.10: Encryption and decryption wheels for parameters $m = 3$, $s = 8$.

The parameter $s = 8$ is readable from both wheels, since the inner $A = 0$ aligns with the outer $I = 8$, and $8 - 0 = 8$.

On the other hand, the parameter $m = 3$ shows up visually in different ways on the two wheels. For the encryption wheel, if we look at spatially adjacent letters on the outer disk, we find they are alphabetically separated by a distance of 3 (with wrapping). For the decryption wheel, if we look for consecutive letters of the alphabet on the inner disk, we will find they are spatially separated by a distance of 3. This latter effect is the most visually apparent, which is fortunate, since users will spend the bulk of their time with the app decrypting.

If we try instead looking at spatially adjacent letters on the inner disk of the decryption wheel or alphabetically consecutive letters on the outer disk of the encryption wheel, we find the separations are instead 9. What we are seeing is a manifestation of $m^{-1} = 9$.

3.2.3 Other ciphers

In a Vigenère cipher, several Caesar ciphers are chosen and used periodically. For example, we could choose $s_1 = 3$ and $s_2 = 8$, in which case we would use $s_1 = 3$ to encrypt the characters in positions $1, 3, 5, 7, 9, \ldots$ and $s_2 = 8$ to encrypt the characters in positions $2, 4, 6, 8, 10, \ldots$. In our app, this is

represented by giving the user several cipher wheels to manipulate.

In a general simple alphabetic substitution cipher, a permutation of the alphabet is chosen. While a crypto wheel is not particularly instructive here, it can provide a visualization of the "randomness" of the permutation.

While each of these ciphers diverges from the family of affine ciphers in its own way, the important aspect is how large the space of each is. For perspective, there are 26 shifts, giving 25 of usable Caesar ciphers. The space of parameters for affine ciphers has size $12 \cdot 26 = 312$, giving 311 usable affine ciphers.

On the other hand, there are

$$26! = 403,291,461,126,605,635,584,000,000$$

permutations of the alphabet. This includes things like the permutation where A and B are swapped but everything else is fixed, so it's a little harder to say which of these are "usable," since some subset is easily decrypted. There are $148,362,637,348,470,135,821,287,825$ derangements of the alphabet (permutations where no letter is fixed), and these would definitely be usable ciphers.

If no bound is given on period, there are infinitely many Vigenère ciphers. If we allow the period to be any number in the range $1 - 10$, then we end up with about

$$26 + 26^2 + 26^3 + \cdots + 26^{10} = 146,813,779,479,510$$

ciphers, before subtracting out duplicate and easily broken ciphers, which make up the minority.

Since the spaces for these ciphers are so much larger, decryption puzzles require longer messages in order to provide the user with enough clues to crack them.

3.3 User response

This activity debuted at the JRMF community math circle on December 17, 2022. The circle was attended by over 60 students. Students were divided into small groups based on their mathematical maturity and led by facilitators.

3.3.1 Impressions

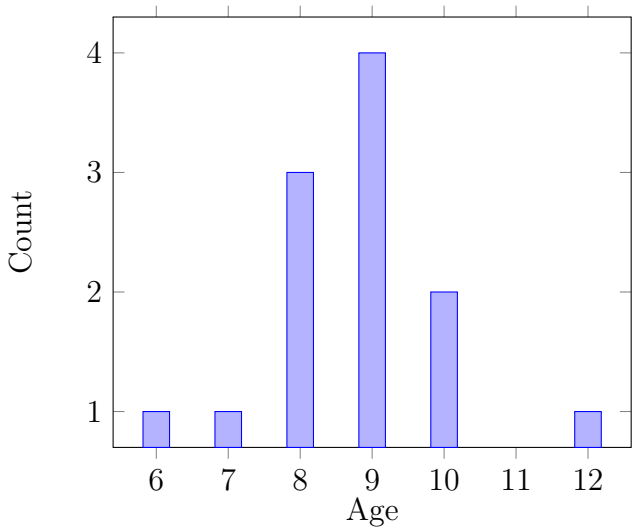
The younger students spent most of their time on the Caesar cipher puzzles, while the intermediate students moved on to substitution ciphers. Counter-intuitively, the advanced students spent most of their time on the Caesar ciphers, but it was mostly because they opted to collaboratively write a Python program to help them brute force the puzzles.

Although many more students attended, only 12 students and 2 adults filled out the emailed follow-up survey. The response was overwhelmingly positive throughout the activity and mostly positive in the survey. Of the negative comments in the survey, most seem to refer to some of the difficulties with doing the activity as a group over Zoom rather than the app itself. In a Zoom window, the text is smaller and much more difficult to read than when running the app full screen, for example. While the facilitators each had their own style of introducing the activity, some feedback indicated that it would be good if there were a more formal tutorial.

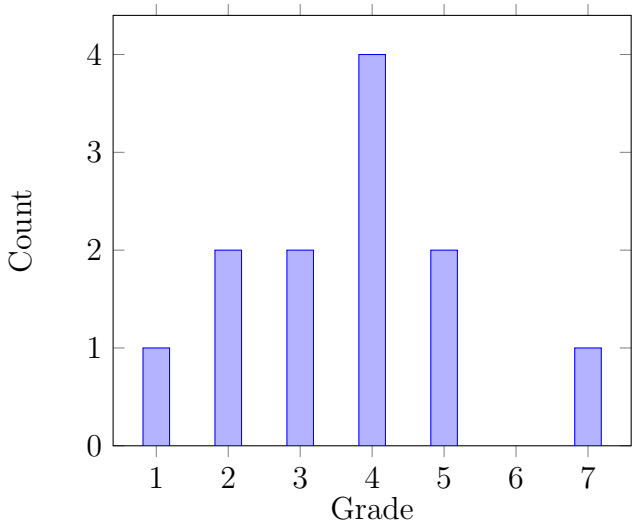
Due to it touching on topics a little closer to standard math enrichment curricula, there is added excitement about this activity's potential. Since people in our community expressed interest in running the activity again as a Zoom activity, we will continue to experiment with element sizing. Aside from a tutorial, the most requested feature was sandbox mode, where students can enter messages, encrypt them using any of the ciphers available, and optionally iteratively reencrypt to allow students to explore the group structure of ciphers.

3.3.2 Survey results

1. What is your age?

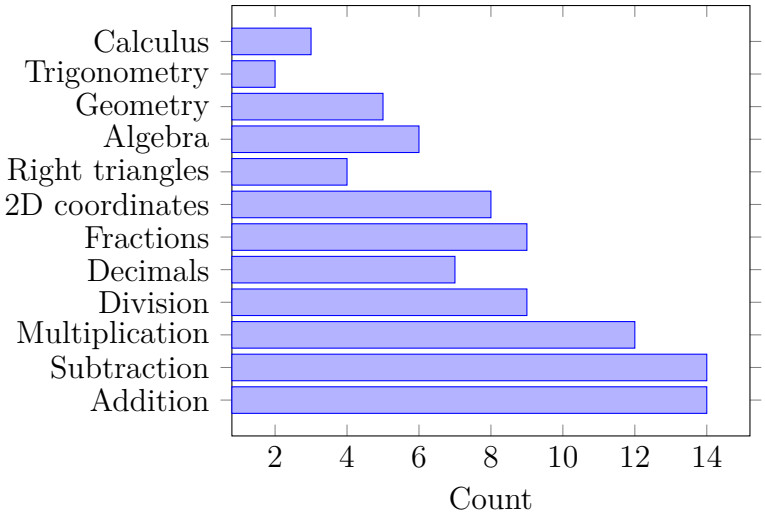


2. If currently in school, what is your current grade or year?

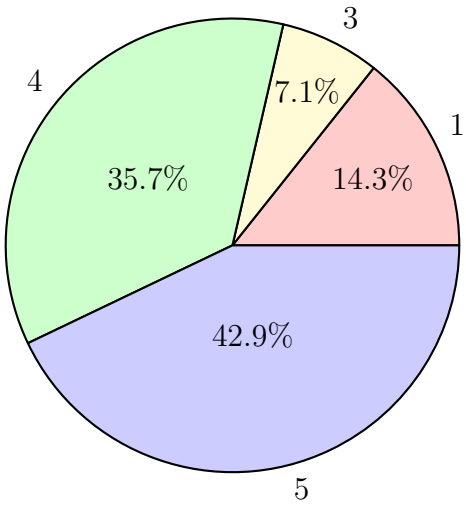


3. Which of these math topics are you somewhat comfortable with? (check all that apply)

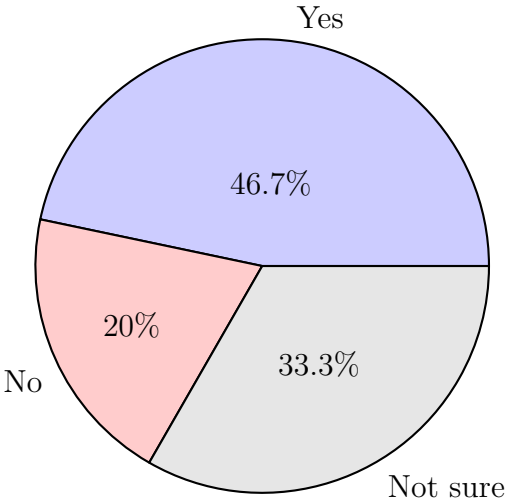
(Adult answers included from this point on.)



4. How much did you enjoy the activity? Scale: 1 (I did not like it at all) to 5 (I liked it a lot)



5. This activity was based on some principles of electronic circuit design, which people do both for fun and for work. Do you think that is something you might like to try?



6. What did you like about the activity?

the puzzles
There were a lot of puzzles I could solve
It was fun
I thought that it was a challenge
Actually on the previous question: 10, I loved it. It was challenging and fun„,
no
Challenging, fun, enjoyable because it's always a different type of code/challenge. I would recommend it.
no
I had fun thinking about ways I could do codes like this with my friends afterwards.
Fun activity, great app to experiment with code shifts
I liked how there were so many puzzles
learned things
I enjoyed learning about this activity. It was new to me.
Fun and educational

7. What didn't you like about the activity?

Nothing!
nothing
Some were unplayable
technical difficulties
Nothing
Somebody kept saying he wanted to do it and didn't allow anybody else to do it on the screen...
the hard part
Some were a little too challenging because some of the codes were so long.
it was hard
This activity was slightly harder to participate over zoom with others. The code wheel was small over video and difficult to see. However, after the session when we played the game ourselves using the app, we could enlarge the code wheel and then everything was super clear and my daughter ended up loving the activity.
was kept in the dark too long
The directions were confusing. The directions were not explained clearly.
None

8. What did you learn from the activity?

the wheel
how to crack puzzles
about Ceaser cryptography
Mods
Testing
nothing
I learned about Caesar codes and ideas/strategies for code cracking.
nuthin
That there are a lot of cool apps out there!
How to try use different techniques to make guesses for initial shifts in the code wheel.
Good strategies
understood Vigenere coding
I learned about a new way of thinking.
Breaking codes with a wheel

9. Do you have any suggestions for improving the activity?

no
No
No not really
Nope it was perfect.
make It easyr
It would be nice if there were a few more puzzles that were shorter, maybe some recognizable ones, like song lyrics perhaps? Or a riddle? The poems: I'm not sure I liked them liked them.
make it easier
No.
Make the code wheel larger in the UI.
start with how to code before guessing a strategy for decoding
I would have enjoyed learning about the puzzle through an example.
None